DOI: 10.1111/cgf.12778

COMPUTER GRAPHICS *forum*
Volume 35 (2016), number 8 pp. 95–107

Visualizing Waypoints-Constrained Origin-Destination Patterns for Massive Transportation Data

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Abstract

Origin-destination (OD) pattern is a highly useful means for transportation research since it summarizes urban dynamics and human mobility. However, existing visual analytics are insufficient for certain OD analytical tasks needed in transport research. For example, transport researchers are interested in path-related movements across congested roads, besides global patterns over the entire domain. Driven by this need, we propose waypoints-constrained OD visual analytics, a new approach for exploring path-related OD patterns in an urban transportation network. First, we use hashing-based query to support interactive filtering of trajectories through user-specified waypoints. Second, we elaborate a set of design principles and rules, and derive a novel unified visual representation called the waypoints-constrained OD view by carefully considering the OD flow presentation, the temporal variation, spatial layout and user interaction. Finally, we demonstrate the effectiveness of our interface with two case studies and expert interviews with five transportation experts.

Keywords: visual analytics, visualization, user interface design, OD pattern, transportation data, waypoints-constrained

ACM CCS: H.5.2 [Information Interfaces and Presentation]: User Interfaces-Evaluation/Methodology I.3.8 [Computer Graphics]: Application-Geographical Visualization

1. Introduction

Origin-destination (OD) pattern is a fundamental concept in transportation, summarizing people and vehicle movements across geographical regions [Voo55]. Studies show that analysing OD patterns can facilitate the understanding of urban dynamics and human activities, for example, estimating region functionality [YZX12], revealing urban structure [JFJG12] and studying congested road usage [WHB*12].

As such, OD pattern has been an important topic in the study of transportation and urban planning. However, visualizing OD patterns has always been challenging. First, considering real transportation data with numerous locations and passenger trajectories, huge amount of OD pairs could be easily produced. As a result, the visualization will likely end up with visual clutter if we simply employ conventional visualization methods like flow map. Second, recent research by Wang *et al.* [WHB*12] shows that only a few (less than 2%) of the road segments in urban areas give rise to congestion. This motivates transportation researchers to study OD patterns subject to specific locations/paths rather than to the entire city. However, existing visual analytic methods generally focus on global OD flows across regions and ignore OD flows constrained along specific locations/paths. Moreover, city-scale OD patterns can be highly complex. That is, the traffic condition in a city could have huge spatial and temporal variations, for example, peak versus non-peak hours, busy versus deserted roads, etc. Lastly, transportation researchers are concerned with not only the OD flow volumes, but also the movement paths of the OD flows.

To address the above spatial-, temporal- and path-related requirements, we design a new visual analytics approach, namely *waypoints-constrained OD visual analytics*, aiming to help users analyse OD patterns associated with trajectories that successively pass through specific links or waypoints in the transportation network. This approach could help transportation researchers in transportation planning and traffic management, for example, in a situation where some subway routes are disrupted, service providers can identify closely connected origins and destinations and provide emergency bus services for commuting the passengers.

Our approach is achieved through an iterative design process. First, we set forth the requirements and analytical tasks in collaboration with the transportation researchers. Second, we elaborate a set of design principles and rules, and carefully consider the OD flow presentation, the temporal variation, spatial layout, and user interaction. when designing the *waypoints-constrained OD view*. Third, we use a hashing-based query method to support interactive filtering with over ~2.1 millions of daily passenger trajectories. Lastly, to demonstrate how our visual analytics interface helps to study and explore the Singapore public transportation data, we present two case studies and conduct an expert review with five transportation researchers.

2. Related Work

2.1. Visual Analytics of Movement Data

Movement data can be modelled as a set of discrete objects whose positions change over time [HE02]. Visual analytics of movement data has been a hot research topic in the visualization community [AA13]. Pioneering methods such as flow maps [Tob87] and space-time cubes [Häg70] were developed to present the spatial and temporal semantics of movement data.

Recently, a number of visualization research works focused on traffic data in urban environments, aiming at facilitating the understanding of traffic patterns at intersections [GWY*11], [ZFMAQ13], suggesting better driving routes [LGL*11], analysing the traffic congestions [WLY*13, WYL*14], exploring the passenger mobility [ZFMA*14] and supporting interactive queries of events [FPV*13, DFD*14]. Among them, Krüger *et al.* [KTW*13] designed trajectory lenses, which can be placed on a map for us to interactively filter movements in a given area of interest while Wang *et al.* [WCW*14] developed a visual analytic interface for selecting and evaluating traffic patterns on specific roads. Compared to our work, these methods are, however, insufficient for supporting path-related and temporal-related OD analysis.

2.2. Visualizing OD data

Flow map, which joins origins and destinations by straight/curved lines, is a common visualization approach for presenting OD data [Tob81, Tob87]. Flow map layout [PXYH05, VBS11] and edge bundling methods [CZQ*08], [HvW09], [EHP*11], [HET12] have been proposed to reduce the visual clutter by bundling paths into smooth splines, and spatial generalization methods [Guo09, AA11] were also proposed to partition underlying territory into appropriate areas and aggregate flows between them.

Another common approach to present OD data is matrix visualization. Since OD data can be modelled as an M-by-N matrix that summarizes flow volumes ($M \cdot N$ OD pairs) from M origins to N destinations, we can sort and re-order the matrix rows and columns to reveal apparent clusters [GCML06]. Ghoniem *et al.* [GFC04] showed that for large/dense graphs, matrix representations are more suitable than node-link views. Wood *et al.* [WDS10, WSD11] further preserved the spatial structure of origins and destinations to improve the OD matrix representation by dividing the geographical domain into regular grids.

Besides, one may also present OD data by showing the mutual relationships between origins and destinations. In general, these techniques often present OD patterns over a period, for example, by animation and small multiples [BBL12]. However, even with an animation, temporal variations are still hard to track and identify [TMB02], while with small multiples, details are still difficult to be observed due to the small display size for each timestamp [BBL12]. To address these issues, Boyandin *et al.* [BBBL11] proposed Flowstrates, which shows origins and destinations on two separate geographical maps (left and right) and presents temporal changes by a heat-map visualization in the middle.

However, these visualization methods could not fulfill our requirements. For flow maps that show straight/curved lines from origins to destinations, they could have visual clutter and normalization issues [GZ14]. For flow map layouts and edge bundling methods, they are suitable mainly for presenting OD flows from one or two origins, or in some special situations [AA13]. Lastly, OD matrix visualization and Flowstrates [BBBL11] are not suitable for our case since they require a considerable amount of screen space to present matrices, which cannot be naturally integrated with other visual elements for showing path and temporal information.

This work focuses on a novel aspect of OD patterns, that is, *waypoints-constrained OD patterns*, which associate with trajectories that successively pass through certain waypoints in an urban transportation network. Compared to previous works, our method allows users to explore OD patterns in an ‘ *overview first, zoom and filter, then details on demand*’ manner [Shn96], and it presents not only origins and destinations, but also their temporal variations and the related trajectory paths in a unified view.

3. Overview

3.1. Transportation Data

The data we employed are from the Singapore Mass Rapid Transit (MRT) system, which is a metro system consisting of about 2.1 million daily passenger trips. In Singapore, passengers carry personalized Radio-frequency Identification (RFID) cards to enter and leave the public transportation system by tapping their own RFID cards on the card readers available in the stations. The card readers can automatically record various trip information such as card ID, tap-in time, tap-out time, related stops, etc. From these raw data, one can reconstruct the passenger trajectory path with time stamps over intermediate stops for every trip record [EFvE*12].

3.2. Basic Concepts

The public transportation network can be represented as a directed graph $G := (V, E)$, where V is a set of nodes in G and E is a set of directed edges connecting neighbouring accessible nodes (locations). Hence, a trajectory T is a sequence of consecutive directed edges i!n G:

$$T := v_1 \to v_2 \to \ldots \to v_m,$$

where $v_i \in V$ and $2 \leq m \leq |V|$. Moreover, we have a timestamp t_i at each v_i along trajectory T. The waypoints-constrained OD pattern associates with trajectories that successively pass through two user-specified waypoints in the transportation network: the *entry waypoint* node, which receives passengers coming from different origins, and the *exit waypoint* node, which sends passengers to their destinations. As a convention, we represent the entry and exit waypoint nodes as red and blue glyphs, respectively, see Figures 1(a) and (b).

3.3. Analytical Tasks

In our collaboration with transportation researchers, we identified a family of analytical tasks. First, our interface should support interactive filtering of trajectories, say $\{T\}$, that successively pass through the user-specified entry and exit waypoints for a given time period. Then, the interface should present spatial- and temporal-related information to support the following basic tasks:

- *T1*: Find the origins and destinations from $\{T\}$;
- *T2*: Examine and compare the flow volumes among the OD pairs derived from $\{T\}$; and
- *T3*: Examine and compare the temporal changes in flow volumes among the OD pairs.

Besides, they would also like to perform some path-related tasks specifically for OD patterns in urban traffic data:

- *T4*: Present the paths through which the trajectories go from origins to destinations; in some situations, we may have multiple paths between the entry and exit waypoints.

3.4. System Overview

Figure 1 shows the workflow. First, the user interactively manipulates the entry and exit waypoints by simply clicking and dragging the red and blue glyphs in the network (Figure 1a). Upon changes in waypoint locations or user-specified time period, the interface automatically filters and queries relevant trajectories from millions of daily trajectories, and presents a map view of the retrieved trajectories in a split second (Figure 1b). Then, the user can bring in the *waypoints-constrained OD view* (Figure 1c) to explore not only the spatial and temporal semantics of the OD patterns but also the path-related information. Note that this *waypoints-constrained OD view* has three main components: *in-flow view*, *OD-flow temporal view* and *out-flow view*, see Section 5 for detail. These components work together to support the analytical tasks mentioned above.

4. Waypoints-Constrained OD Query

4.1. Interactive waypoints specification

To construct a visual query, users can interactively specify and manipulate the (entry and exit) waypoints by simple mouse click and drag with the following three modes:

Mode 1: two successive mouse clicks to select an entry waypoint and then an exit waypoint, say A and D in Figure 2(a). Our interface then applies the query method in Section 4.2 to retrieve all relevant trajectories through A and then D by considering all possible time-efficient paths from A to D, that is, $A \to B \to D$ and $A \to C \to D$.

Mode 2: click to select an entry waypoint (A) and then drag along the network to define a path (ABD), see Figure 2(b). The node at which the mouse button is released defines the exit waypoint (D), and we consider only the trajectories along the dragged path $A \to B \to D$ but not $A \to C \to D$.

Mode 3: long-press to select a common entry and exit waypoint, say A in Figure 2(c). The interface then shows all outgoing (blue) arrows emerged from A. In this mode, the user can select different outgoing directions from A and explore the OD patterns of trajectories along different directions from the same junction node.

Once the waypoints are specified, the user can interactively modify them on the map, see Figure 2(d). In case the entry and exit waypoints become coincident, the query mode smoothly changes from mode 1/2 to mode 3, and vice versa.

4.2. Hashing-based trajectory query

To support such interactive query, we need to efficiently filter out relevant trajectories against a given time period ($\triangle t$). Rather than scanning through the nodes along every trajectory in the data, we use a hashing-based method. First, we attach a unique ID (tid) to each trajectory, and define 72 equal time intervals from 6 A.M. till midnight, each covering 15 min. The choice of this interval size is driven by a common practice by transportation researchers: when modelling and analysing transport data, they normally set a minimum analysis time interval, which should not be too short, so that there are sufficient samples in each interval, and should not be too long to avoid losing the details. Altogether, there are two stages in the query method:

Indexing Scheme (offline). First, for each edge $e := < v_i, v_j >$ in G, we record per time interval a list of tid of trajectories that pass through e. A trajectory T is said to pass through e within $\triangle t$ if T's $[t_i, t_j]$ overlaps $\triangle t$, where t_i and t_j are time at which T passes through v_i and v_j of e, respectively.

Second, we build a hash table for each node v in G, where the hash key is a tid and the hash value is the corresponding time at which the trajectory passes through v. Since hashing can be done in $O(1)$ time, we can quickly check if a given trajectory (tid) passes through v, and if this is the case, we can also obtain the related timestamp (t_v).

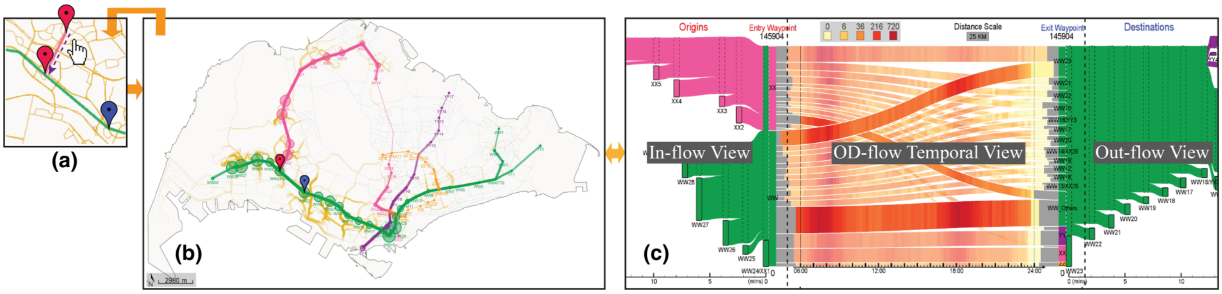

Figure 1: *Overview of our waypoints-constrained OD visual analytics interface. We can interactively (a) select and modify the entry (red) and exit (blue) waypoints in the transportation network, (b) filter passenger trajectories that successively pass through the selected waypoints and (c) present the waypoints-constrained OD view to support the analytical tasks.*

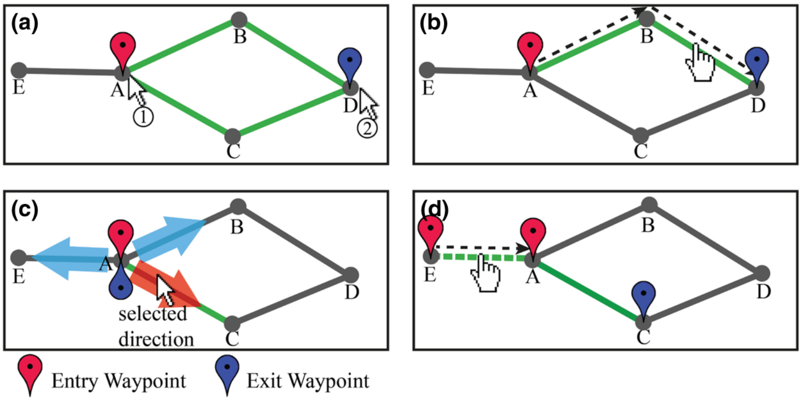

Figure 2: *Interactive waypoints specification: (a) mode 1: two successive clicks to select an entry and then an exit waypoint; (b) mode 2: drag to define a path from the entry waypoint to exit waypoint; (c) mode 3: click to select an entry waypoint and an outgoing direction (red arrow); and (d) click to select and then drag to modify an existing waypoint.*

Trajectory Query (online). Given entry waypoint A, exit waypoint D and time interval $\triangle t$, our interface performs the following two steps to extract the relevant trajectories.

In the first step, our goal is to determine a set of candidate trajectories (i.e. a list of $tid's$), say S_A, that exit from A within $\triangle t$. After identifying A's outgoing edges that are relevant to the query, we retrieve and combine the lists of pre-computed tid's through each edge for the 15-min time interval(s) covered by $\triangle t$. Note that if $\triangle t$ covers more than one 15-min time interval, we have to remove duplicated tid's when combining the pre-computed lists since some trajectories may appear in two consecutive time intervals. It is because when a passenger travels through a link (edge) in the transport network, he/she may start the travel within a certain time interval and end it within the next time interval.

In the second step, our goal is to filter the trajectories in S_A, and output those that pass through D within $\triangle t$. Here, we employ the pre-computed hash tables for speedup. In detail, we first retrieve the hash table at exit waypoint D, and use it to remove the trajectories (in S_A) that do not pass through D, or pass through D but outside $\triangle t$ (by the hash value). In case of mode 2, we need to perform this test for every intermediate node along the user-selected path from A to D.

After these tests, the remaining trajectories are the query result for constructing the OD flows. Note also that in case of mode 3 (with same entry and exit waypoints), we can skip the second step and output S_A as the result.

4.3. Map view

After the query, we present a map view (Figure 3) to overview the amount of retrieved trajectories over different segments in the transportation network. This map view provides intuitive spatial information essential for locating the origins and destinations in the physical space (*Task T1*).

This map view is created by hardware rendering. First, we randomly jitter the position of each trajectory by a few pixels and render each of them with low transparency, so the resulting plotting effect can roughly reveal the flow volume. Note also that we follow the Singapore MRT (metro) colouring scheme to colour different parts of the trajectories, for example, green for WW line and red for XX line.

Besides, the map view also presents: *Entry and Exit Waypoints* as red and blue glyphs, respectively, on the map, see Figure 3(b).

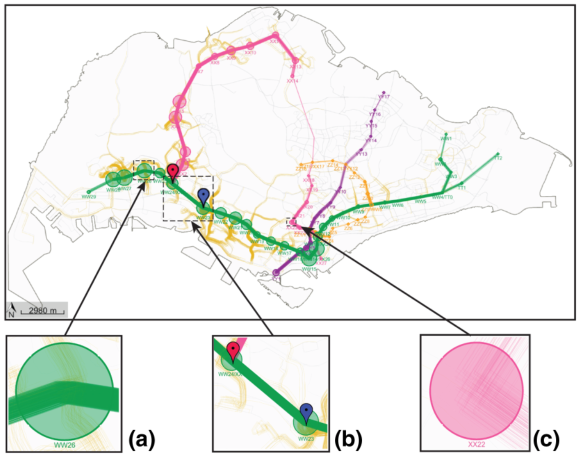

Figure 3: *The map view overviews the waypoints-constrained trajectories: (a) an origin, (b) red and blue glyphs as entry and exit waypoints, resp., and (c) a destination.*

Origins and Destinations as hollow circles positioned at their locations with radii revealing the corresponding flow volume, see the hollow circles in Figures 3(a) and (c).

5. Waypoints-Constrained OD View

In this section, we first discuss the design philosophy behind our interface. Then, we elaborate the three component views in the waypoints-constrained OD view, and discuss alternative designs. Lastly, we present the layout algorithm and user interaction we developed in the interface.

5.1. Design philosophy

By the map view, we can present aggregated flow volumes across origins and destinations, and support analytical task *T1*. However, it is clear that the map view alone is insufficient for other tasks: *T2* to *T4*, for example, Figure 3 does not allow us to examine and compare flow volumes among different OD pairs from WW28 (Figure 3a), so it cannot handle *T2*.

This calls for a new visualization design to address tasks *T2* to *T4*. In particular, we aim to depict the OD pairs (*T2*), the temporal-related information (*T3*), and the path-related information (*T4*) in an integrated and coordinated fashion. This is a very challenging problem: (i) visual clutter could easily occur given excessive OD pairs; (ii) integrating temporal-related information of OD pairs in the visualization could further increase the visual clutter; and (iii) it is non-trivial to also support the tracing of flow paths from origins to destinations. To address these challenges, we identify the following principles as guidance of our design:

- **Overview+Detail.** The visualization should support an overview of the OD patterns, for example, appropriately summarizing the origins and destinations, in order to address the visual clutter issue when presenting the OD pairs. Certain interactive exploration should also be incorporated in the design to allow users to further analyse the OD patterns with controllable amount of details on demand.
- **Visual Correlation with Transport Semantics.** The visualization should reveal the semantics of the transport network to promote the correlation between elements in the visualization and the actual objects they represent, for example, MRT stations and lines. Since the visualization has to include OD pairs, temporal- and path-related information, we cannot directly put the design on the map view.
- **Intuitive Spatial Layout.** The spatial layout in the visualization should be intuitive for users to explore the origins, the destinations, and the OD flows in-between them. Furthermore, the spatial layout should facilitate intuitive user interaction needed by the users.

By considering the above design principles, we formulate a novel visual design, namely the *waypoints-constrained OD view* with three component views: in-flow, OD-flow temporal and out-flow views (Figures 4a–c). In particular, we establish the following convention rules to meet the '*visual correlation*' and '*spatial layout*' principles:

First, we separate the three component views by two vertical bars (Figure 4) that represent the entry and exit waypoints: origins on the left (in-flow view), destinations on the right (in-flow view), whereas the connections between OD pairs are in the middle (OD-flow temporal view). Hence, this integrated design can naturally present the OD flows, which generally go from left to right. Second, the vertical dimension in the three-component views always indicates the flow volume. Third, we always try to use the standard Singapore MRT colouring scheme for the visual elements in the visualization since this helps reveal the semantics of the transport network, for example, green for the WW line, red for the XX line, purple for the YY line and yellow for the ZZ line.

5.2. The in-flow and out-flow views

These two views are designed for users to effectively compare flow volumes of different stations (part of *Task T1*) and visualize the paths from origins to destinations (*Task T4*).

First, origins and destinations are presented as solid rectangular boxes; their colours follow the Singapore MRT colouring scheme, for example, the green and red boxes in Figure 5, while their heights indicate the associated flow volume. Second, the horizontal axis in both views (Figure 5, bottom) indicate the travel time from origins to entry waypoint (in-flow view), or from exit waypoint to destinations (out-flow view). So, we can easily observe and compare the travel time between different origins/destinations and the two waypoints. Third, we adopt the Sankey flow diagram to connect the boxes and their associated waypoint with smooth ribbons whose heights indicate the (aggregated) flow volume.

From the in-flow view in Figure 5, we can see that stations WW23 and WW27 contribute the most in terms of flow volume to entry waypoint WW22. Besides, flows from the XX line merge into the green WW line at station WW24/XX1, indicating that passengers from the XX line transfer to WW line at WW24/XX1. Here, different portions of height vertically along station WW24/XX1 reveal the aggregated flow volumes from the XX line, from the WW line

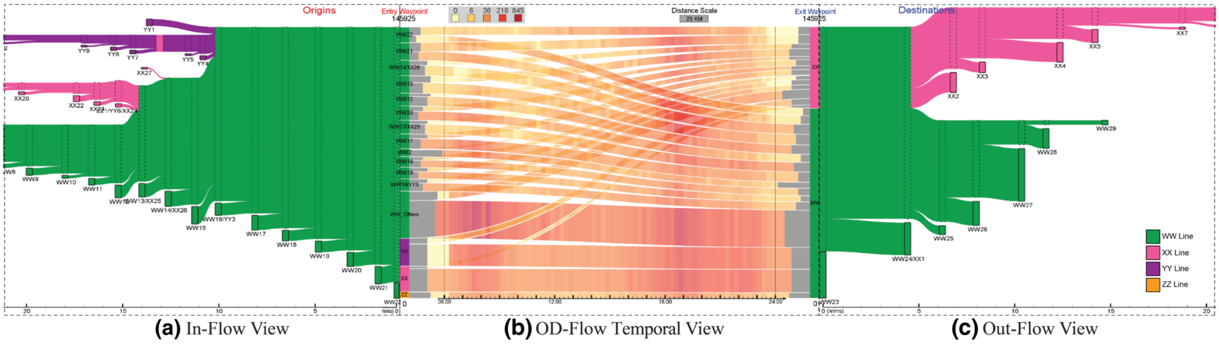

Figure 4: *An example waypoints-constrained OD view with three components: (a) in-flow view, (b) OD-flow temporal view, and (c) out-flow view, as an integrated and coordinated solution to support the visual analytical tasks.*

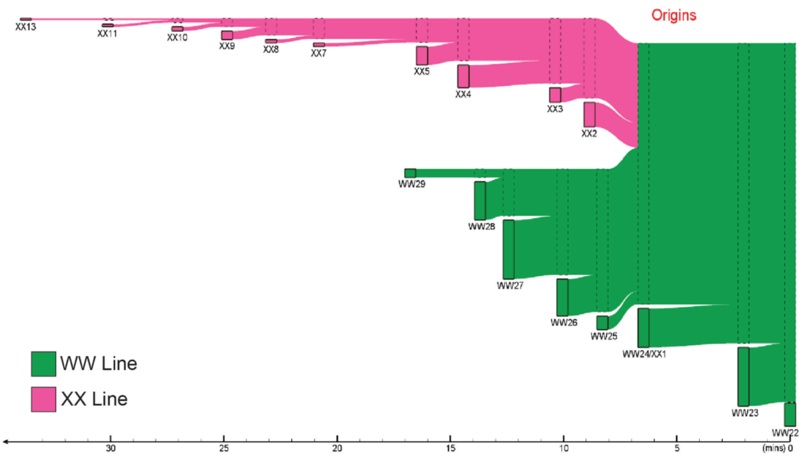

Figure 5: *An example in-flow view: vertical boxes show the origins with heights to indicate flow volumes, and ribbons to show flow aggregation before the entry waypoint.*

(WW25 to WW29), and from WW24/XX1 itself. Note that in Singapore, interchange stations connecting multiple MRT lines compose of multiple IDs, for example, WW24/XX1 is the 24th station along the WW line and 1st station along the XX line.

5.3. The OD-flow temporal view

The OD-flow temporal view (Figure 6) is designed to support *Task T2* & *Task T3* with the following visual elements:

1. *Origins and Destinations.* The two long vertical bars represent the entry and exit waypoints (Figure 6a) and contain boxes that represent the origins and destinations. These boxes can be manipulated by users for '*overview+detail*' exploration, that is, users can overview the OD flows by aggregating the origin/destination boxes, or explore their details by decomposing them, see Section 5.6. Note also that we group the boxes by MRT lines and sort them by the method in Section 5.5.
2. *OD-Pair Flow.* To address *Task T2*, we use smooth ribbons (Figure 6b) to connect the origin and destination nodes (boxes) to show the flows for each OD pair over the given time period shown on the bottom of the view. Here, we emphasize the OD flows with larger flow volumes by rendering the ribbons from back to front in ascending order of flow volumes, and we add halos around the ribbon boundaries to help reveal the layering.
3. *Temporal Variation of OD-Pair Flow.* To address *Task T3*, we adopt a heat map visualization on each ribbon to present the associated temporal variation of flow volume over the given time period. In detail, we horizontally divide each ribbon into column segments, each corresponding to a 15-min time interval along the horizontal time axis on the bottom. Each column segment is then coloured based on the colour map shown on top of the view.
   We consider two mechanisms to render overlapping ribbons in the OD-flow temporal view. By default, we employ translucency to blend these ribbons since this approach can preserve the visual connection and continuity of ribbons, enhancing the tracing of ribbons on the back layers. Other than that, users can optionally make the ribbons opaque, so that interested ribbons can be highlighted, for example, the front-most ribbons in Figure 6.
4. *OD-Pair Travel Time/Distance.* Optionally, users can examine and compare the relative (average) travel time/distance among the OD pairs by looking at the grey segments at the ends of the ribbons near the two vertical bars. For the case shown in

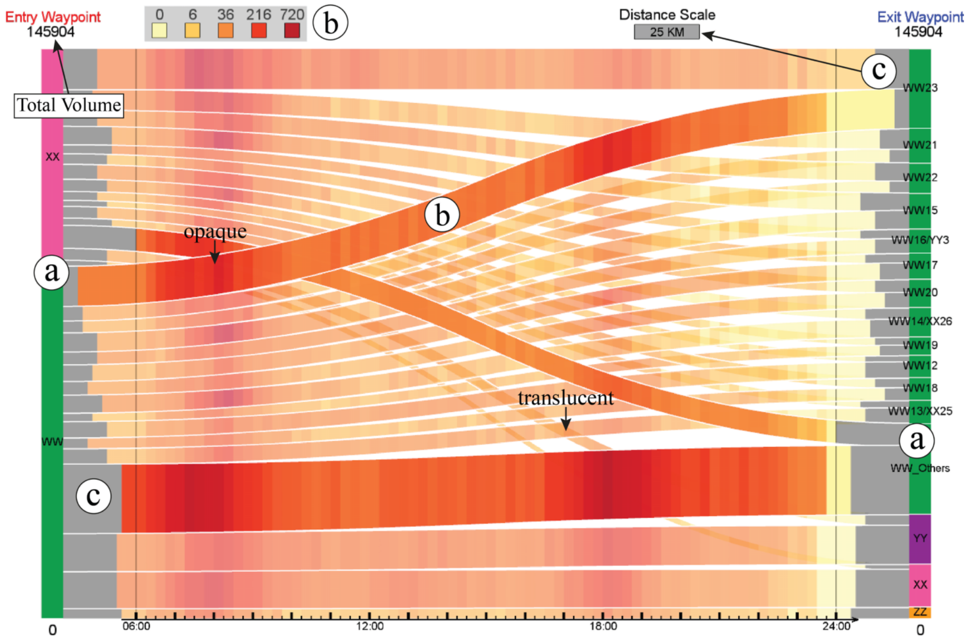

Figure 6: *An example OD-flow temporal view that presents the OD patterns for the trajectories shown in Figure 3: (a) the two vertical bars represent the origin waypoint (left) and destination waypoint (right), and the coloured boxes along the two bars represent the corresponding origins and destinations, respectively, (b) ribbons connect the origin and destination boxes and show the OD flow volumes in-between the corresponding OD pairs with embedded heat maps to reveal flow volume variation over time, and (c) relative (average) travel distance of each OD pair. Note also that the heat maps are coloured by the colour bar on top, for example, the rightmost deep red is used for flow volume in range* (216, 720].

Figure 6(c), the grey segments show the travel distance of the OD pairs, see also the time/distance scale mark on top of the view for facilitating the visual examination and comparison.

5. *Multiple Inter-Waypoints Paths.* In some situations, we could have multiple time-efficient paths in between the entry and exit waypoints, for example, the two green paths in Figure 2(a). In this case, we put in additional vertical boxes in the middle of the OD-flow temporal view to bundle the OD flows across different time-efficient paths (see the green and purple bars in the middle of Figures 9a and b). This complements the in- and out-flow views to help users explore the path details.

Figure 6 shows an example OD-flow temporal view for the trajectories that pass through the entry waypoint (station WW24/XX1) and the exit waypoint (station WW23) during the time period 6:00 to 24:00. The origin WW stations (Figure 6a, left) are aggregated as one single group, while the destination WW stations (Figure 6a, right) are disaggregated as 12 single stations and one sub-group. The visualization here shows that there is slightly higher incoming flows originated from the *WW* stations than from the *XX* stations, and most trajectories end at *WW* stations. Moreover, we can observe the morning and evening peak patterns by looking at the heat maps embedded on the ribbons.

5.4. Discussion: design alternatives

This subsection discusses design alternatives for *T2* to *T4*:

To support task *T2*, which focuses on flow volumes among OD pairs, we connect origins and destinations with Sankey-style ribbons (Figure 4b). Such a design allows for hierarchical clustering of nodes with different details, thus promoting the '*overview+detail*' principle, yet most existing OD visualization methods do not exhibit this flexibility.

To support task *T3*, which focuses on temporal changes of flow volumes, we embed heat maps along the ribbons in the OD-flow temporal view (Figure 4b). Heat map is a widely-accepted and well-recognized method for depicting variations of data over time, and has been applied and proven to be efficient in visualizing temporal OD data [BBBL11].

Typical alternative designs are small multiples and animations. Small multiples can be constructed by putting together multiple OD-flow temporal views, each for a different time interval, see Figure 7. However, due to the small size of individual views, it is difficult to examine and compare flow volumes of OD pairs across different time periods. For instance, we can intuitively observe morning and evening peak flows in Figure 4(b) (with 72 time periods) but not in Figure 7 (with only six periods, when occupying a similar size as Figure 4b in the paper). On the other hand, animations have been shown to be useful in some cases, but they are not effective for supporting statistical analysis [RFF*08].

To support task *T4*, that is, to present the paths through which the trajectories go from origins to destinations, our visualization

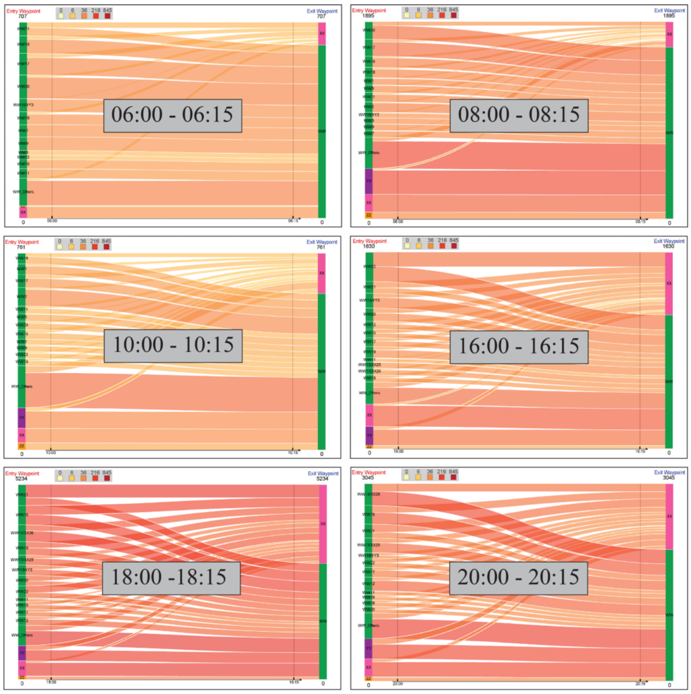

Figure 7: *Design alternative: small multiples that show flow volume difference over six different time periods with the same entry and exit waypoints as in Figure 4. The individual diagrams are too small to present useful information.*

adopts Sankey-style ribbons to support quantitative flow tracing across trajectory paths [RHF05], see Figures 4(a) and (c).

Existing OD visualizations can be summarized into three categories based on the amount of path-related information they present: (i) absent flow paths, for example, OD matrix visualizations without any intermediate location [GCML06], [WDS10]; (ii) discontinuous flow paths, for example, flow maps with arrows connecting neighboring locations [Guo09], [AA11]; (iii) flow paths that are continuous but not existing in reality, for example, bundled flow maps [PXYH05], [VBS11]. None of these methods can support well the flow tracing from *multiple* origins to *multiple* destinations. Compared to these methods, our in-flow and out-flow views, which integrate with the OD-flow temporal view, can intuitively reveal how the flows aggregate before reaching the entry waypoint and how the flows distribute after leaving the exit waypoint.

5.5. Ordering of origin/destination boxes

The readability of the OD-flow temporal view (see again Figure 6) is highly affected by the visual clutter among the ribbons. This problem is commonly found in node-link graphs, where several edge crossings reduction algorithms [HvW08, CBW15] have been proposed to reduce the visual clutter by repositioning the nodes. Here, we adopt the randomized method in [VBAW15] to reorder the origin and destination boxes on the left and right vertical bars, but instead of reducing the number of edge crossings, we aim to minimize the overlapping area among the ribbons since the ribbons in our visualization are much wider than the edges in conventional node-link graphs.

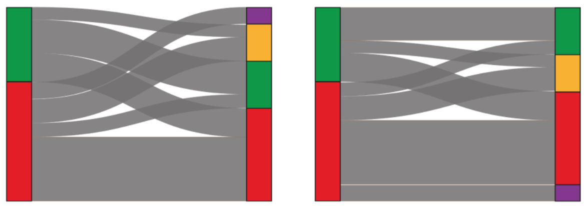

Figure 8: *Illustration: effects of ordering origin and destination boxes by flow volumes (left) and by overlapping minimization (right) in the OD-flow temporal view.*

In detail, our heuristic method starts by ordering the origin and destination boxes in ascending order of flow volumes, see Figure 8(left). Then, we randomly shuffle the boxes on each vertical bar until the total overlapping area cannot be further reduced. Lastly, we select the solution with least total overlapping area. Note also that we keep the grouping of origin and destination boxes based on MRT lines, so the re-ordering is applied only among groups or nodes with the same parent in the hierarchy. Figure 8(right) presents a typical result, showing that the total overlapping area (dark grey) can be effectively reduced by our method.

5.6. User interaction

Furthermore, we offer a set of user interaction methods to facilitate '*overview+detail*' exploration of OD patterns apart from waypoints specification and time period selection:

Aggregate/Disaggregate. the origins/destinations that belong to the same group, for example, the same MRT service line. This allows users to interactively control the number (and details) of OD pairs to be displayed and to explore the OD pairs at different levels of detail on demand.

Filter. the origins/destinations and their corresponding OD-pairs, enabling users to remove less interested OD pairs and concentrate on the remaining OD flows.

Highlight. the OD pairs by selecting and emphasizing interested ribbons. This action brings the selected ribbons forward in the layering order and makes their colours opaque (see the front-most ribbons in Figure 6b).

6. Evaluation and Discussion

6.1. Performance evaluation: trajectory query

We evaluated the performance of the hashing-based trajectory query method on a PC with a dual Intel(R) Xeon(R) E5-1650 CPU and 16 GB memory, and used the Singapore MRT transport data with two million passenger trajectories on a typical working day. The trajectories contain 8.3 stops on average from origin to destination, so the whole data contains around 16.5 million trajectory nodes in total.

In this experiment, we compare the performance of our method against a simple method that sequentially looks through all the nodes along each trajectory to find out the trajectories that pass through the two waypoints within a given query time period. Here, we randomly picked 10%, 50%, and 100% of the trajectories from the whole data set to form three data sets, and performed three tests on each of them with different query time periods: 15 min, 1 h and 24 h, respectively. In each test, we randomly generate 5000 pairs of entry and exit waypoints and record the query time for each pair. We obtained the following results:

- The query time of both methods increase (apparently linear) with the number of trajectories in the data set, but only our method is affected by the length of the query time period. The performance of our method is also affected by the entry waypoint and the query time: it takes longer query time with a busy entry waypoint (with numerous trajectories) or with peak hours.
- In the worst case (100% trajectories and 24-h query time period), our method finishes in ~45.3 ms on average, while the simple method needs ~8.23 s. Note that this performance is necessary to support interactive query of trajectories when users manipulate the entry and exit waypoints or change the query time period.

6.2. Study 1: transportation network usage analysis

In study 1, we aim to analyse and explore flows among OD pairs, temporal- and path-related network usage, which are mainly related to tasks *T2*, *T3* and *T4*, respectively.

Here, we specify two time periods, 06:00-10:00 (Figure 9a) and 16:00-20:00 (Figure 9b), to compare the network usage for the same pair of waypoints: MRT stations WW16/YY3 and ZZ1/YY6/XX24. There are two time-efficient paths in-between them: the green (WW) and purple (YY) paths, see the map view in Figure 9(c). Correspondingly, we put two vertical boxes in the middle of the two visualizations to represent the two paths (or branches).

To address task *T2* in this case study, we highlight and compare the ribbons of two different OD-pair flows in Figures 9(a) and (b): one starts from WW27 while the other starts from YY1, and both end at the XX line, see Figures 9(d) and (e). From Figure 9(a), we can see that WW27 contributes a slightly larger flow volume to stations in the XX line than YY1 in the morning; while from Figure 9(b), we can see that the ribbon from YY1 to XX line becomes much wider than that from WW27 to XX line, indicating more people coming from YY1 to XX line in the evening.

Our visualization can also support task *T3*. At a glimpse, we can see that Figure 9(a) contains less red colours, indicating less flows through the two waypoints in the morning than in the evening. Second, when further examining the two figures, we can see that the colours around 08:00 in Figure 9(a) and around 18:30 in Figure 9(b) are more dark red than others. This reveals 08:00 as the morning peak time and 18:30 as the evening peak time for flows across the selected waypoints. Moreover, looking at the height of YY1 in Figure 9(b, bottom left), we can see that it has more flows as compared to other origin stations. This may be due to the fact that YY1 is a popular shopping area near Sentosa in Singapore; more people return home from YY1 in the evening.

Path-related information can also be explored in our visualizations for supporting task *T4*. From Figure 9, we can find that people choose the intermediate paths (WW or YY) mainly depending on their origins and destinations. Taking the flows from WW27 to XX stations as an example, that is, Figure 9(d), almost all passengers chose the WW path instead of the YY path, but they chose the YY path if their destinations in the YY stations. Lastly, Figure 9(b) also shows that there are relatively more flows through the YY line than the WW line in the evening, and most of these flows either start from YY1 or end at the stations in the YY line.

6.3. Study 2: daily pendulum movements exploration

In transportation, pendulum movements describe an obligatory urban mobility pattern that is highly predictable and recurring on a regular basis [RCS09]: Employees who commute from residential to working areas contribute to the A.M. peak flow; when they return home, they contribute to the P.M. peak flow. By analysing this pattern, researchers can effectively measure the distribution of residential and business regions [LBR09]. Thus, exploring pendulum movements is highly valuable for transportation planning.

The pendulum movement pattern is mainly related to analytical task *T1*, since we need to determine the locations of the origins and destinations, and then check whether they swap roles in the morning and evening periods. In this study, we explore the daily pendulum movement patterns in the Singapore MRT data on a normal working day.

Here, we first specify stations WW24/XX1 and WW23 as the entry and exit waypoints (red arrow in Figure 10c) and produce

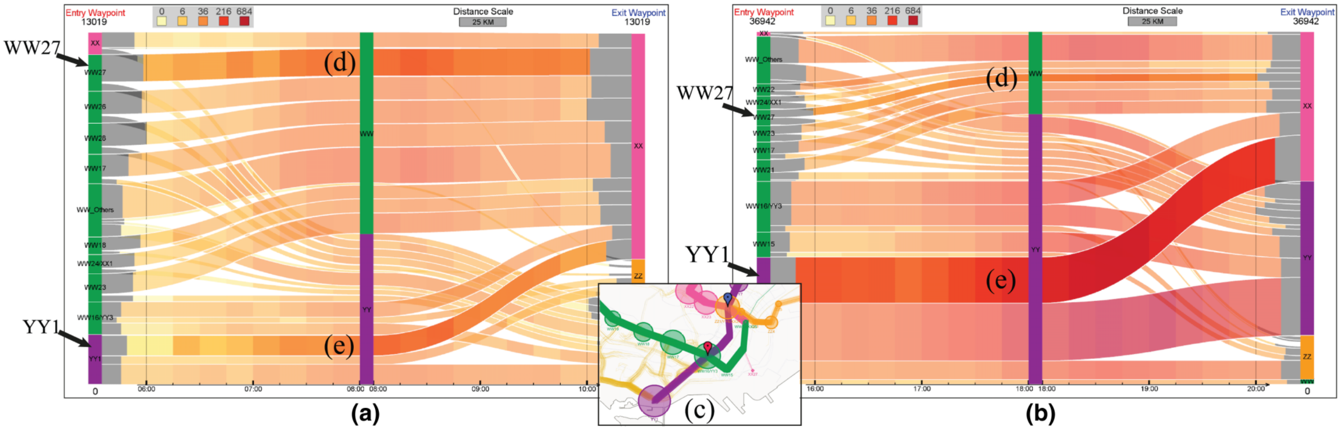

Figure 9: *Case study 1: transportation network usage analysis. (a) & (b) present the morning and evening OD-flow temporal views, respectively, for the same pair of waypoints shown in the map view (c). Highlighted ribbon (d) shows that trajectories leaving WW27 station for XX-line stations (red) always pass through the WW branch (green) instead of the YY branch (purple) (note: they are two different time-efficient paths between the waypoints, see again the map view), while (e) shows that trajectories transferred from YY1 station to XX stations always pass through the YY branch.*

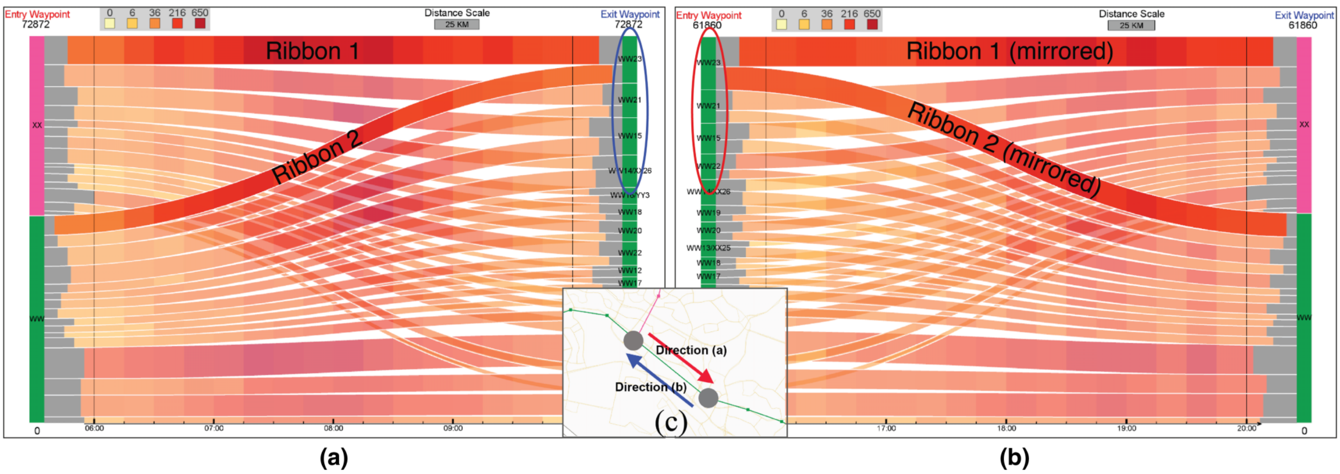

Figure 10: *Case study 2: daily pendulum movement exploration. (a) and (b) present an interesting pendulum movement pattern, which illustrates home-to-work movements through the red arrow in (c) in the morning (06:00-10:00), and work-to-home movements through the blue arrow in (c) in the evening (16:00-20:00).*

the waypoints-constrained OD view for the morning period 06:00–10:00 (Figure 10a). After that, we swap the role of the two waypoints (blue arrow in Figure 10c) and produce another view for the evening period 16:00–20:00 (Figure 10b). By comparing Figures 10(a) and (b), we can observe interesting pendulum movement patterns.

First, both views identify nearly the same set of origins and destinations but with swapped roles. In the morning, WW and XX stations generate similar amount of flows, indicating that the areas around both sets of stations have similar residential population. Moreover, most of these flows end at four specific stations, see the blue circle in Figure 10(a), indicating that these stations mostly locate in business areas as compared to others. Furthermore, while in the evening, most of the flows from the four stations (see the red circle in Figure 10b) have similar flow volumes, mirroring the flows in the morning, suggesting that most people follow reversed routes to return home, so these flows mostly end at WW and XX stations with similar flow volumes.

Second, flow volumes between the same OD pairs in A.M. and P.M. with reversed directions are almost the same. Taking the highlighted ribbons connecting XX-line to WW23 (*Ribbon 1* in Figure 10a) and WW-line to WW23 (*Ribbon 2* in Figure 10a) as examples, we can find similar flow volumes for each ribbon as compared to its mirrored counterpart in Figure 10(b). This further shows that most employees return home from workplace through reversed routes. It would be interesting to explore whether such a pattern also happen in other big cities such as London and New York.

6.4. Expert interview

We conducted expert interviews with five transportation experts: two senior researchers with 15+ years of research experience (denoted as SR1 & SR2) and three junior researchers with less experience (denoted as R1, R2 & R3). Since this research work is conducted through a transdisciplinary research programme (Future Cities Laboratory), which comprises computer scientists, transportation researchers, architects, etc., the first author can easily reach out to transportation researchers in the institute. Here, SR1 is one of the co-author of this paper while the other experts are independent researchers from the institute.

In the expert interview, we started with a few questions to identify their background and explained our interface design and visual encodings. We then showed the two case studies and asked for their feedbacks. Each interview lasted for 1–1.5 h, and their feedbacks are summarized as follows.

Visual design and interactions. In general, all experts agreed that our visual analytics interface is nicely designed, and supports the analytical tasks well. They especially liked the OD-flow temporal view since it can help to reveal both the OD flows over the whole time period as well as in a specific time interval. Normally, they employ conventional flow maps that connect origins and destinations when studying OD patterns. They pointed out that the conventional flow map could easily cause visual clutter with that many OD pairs and that they have to produce many views to compare the ODs at different time periods. SR2 said '*I never thought one single view can clearly present the OD flows and their temporal variations*'. SR1 specially appreciated the order of OD-pair ribbons with larger volumes in front, as '*in general, OD pairs with larger volumes are more interesting*' to them.

The experts also acknowledged the usefulness of the in- and out-flow views as visual aids for exploring the trajectory paths. R3 commented the views: '*intuitively demonstrate the passengers accumulate and spread along the network*'. SR1 & R3 pointed out that being able to observe the travel time from each origin/destination to the entry/exit waypoint is very useful, as it reveals passenger's preference regarding travel time. '*There must be something behind if many passengers need to travel long times*', said SR1.

The experts appreciated the interactions offered by our interface. There can be easily hundreds of OD pairs in OD analysis. Being able to *filter* unimportant and *highlight* important information '*would greatly facilitate my analysis*', said R2. The experts also liked the aggregate/disaggregate interactions, which can reduce the number of OD pairs, allowing them to explore particular ODs on their demand.

Suggestions. The experts gave some fruitful comments to improve our interface. They mentioned that we can provide more spatial information in the waypoints-constrained OD view, such as a dimmed map on the background. SR1 & SR2 also hoped that our system would support some in-depth analysis of some mobility information. For example, they would like to explore if passenger travel distances follow the power-law distribution [BHG06], yet our visualization can only present relative (average) travel distances among the OD pairs. The experts also had some concerns about adopting our interface to more complex subway systems, which do not come with a simple colour coding scheme. Nevertheless, they agreed if we can pre-define subway line colours and do some training, the users would get used to our system.

6.5. Discussion

In this work, we explored waypoints-constrained OD patterns of passenger flows in the Singapore MRT network. Case study 1 demonstrated that our visualization can effectively present OD-pair flows, temporal- and path-related information, with respect to analytical tasks *T2*, *T3* and *T4*, and case study 2 showed that our design can also support well analytical task *T1*. The expert feedbacks further commented the effectiveness of our visual analytic system.

We believe that our system can be adopted to visualize OD patterns of movement data in more complex networks: For example, in a general case, we can hierarchically partition the geographical space into regions based on administrations or methods like [Guo09], [AA11], and explore OD flows in-between these regions. The experts also highlighted that geographical partitioning method could be aligned with their traditional OD analysis.

The OD-flow temporal view depicts large amount of information, yet the design may fail when given excessive quantity of OD pairs. Due to the number of resulting ribbon crossings, exploring OD patterns with more than forty OD pairs altogether (without hierarchical grouping) is not recommended. Nevertheless, our visualization design is suitable for waypoints-constrained OD pattern analysis for two reasons. First, according to previous study [WHB*12], the number of origins and destinations that are interested to transportation researchers is limited in most situations. Second, we adopt the '$overview + detail$' principle, allowing users to interactively control and manipulate OD pairs being presented. Most existing OD visualization methods, for example, Flowstrates [BBBL11], do not offer this feasibility.

Not being able to preserve more spatial context can be considered as a limitation of our approach. However, this is a common problem for methods that visualize arbitrary OD flows [AA13]. To mitigate the spatial information loss, we offer the in-flow and out-flow views to facilitate the exploration of trajectory paths.

7. Conclusion and Future Work

In this paper, we present a novel visual analytics approach, namely the *waypoints-constrained OD visual analytics*. By using a pair of user-specified entry and exit waypoints, which can be interactively manipulated over the map view, we can define visual query to explore waypoints-constrained trajectories with the help of a hashing-based method. After that, we develop the *waypoints-constrained OD view* (the in-flow view, OD-flow temporal view, and out-flow view) to present spatial-, temporal- and path-related information of the OD patterns based on various design considerations. In the end, we performed also two case studies using the Singapore public transportation data with ∼2.1 million passenger trajectories, and conducted an interview with five transportation researchers to examine our visual analytics method.

We believe that there are no universal solutions that can effectively visualize arbitrary aspects of OD flows [AA13]. Our solution is still weak in presenting spatial and temporal OD patterns in a unified manner; this remains a very challenging problem in OD flow visualization. In our collaboration with transportation researchers, we also realize that in-depth understanding of domain-specific tasks, for example, daily pendulum movement pattern, could significantly help improve the effectiveness of the visualization.

In the future, we plan to integrate more visual analytics elements, for example, detailed distance and velocity views as in [WCW*14]. Second, we plan to explore other forms of origins and destinations grouping, for example, by administrative regions. Third, we would like to study multiple pieces of transportation data over years, and analyze calendar patterns in the OD flows to explore the effect of recent upgrades in the transport system. Similarly, we would like to explore difference/change in OD flow patterns over time, for example, in the daily pendulum movement situation. Lastly, as the number of trajectories increases, we plan to explore GPU computing to further improve the performance of trajectory query.

Acknowledgements

We thank all reviewers for their effort and constructive comments, and the transportation experts who provided us with the interviews. This work was mainly established at the Singapore-ETH Centre for Global Environmental Sustainability (SEC), co-funded in part by the Singapore National Research Foundation (NRF) and ETH Zurich, and the MOE Tier-2 fund (MOE2011-T2-2-041 (ARC 5/12)). Huamin Qu is supported by a grant from MSRA.

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Supporting Information

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